USE OF HEAT PIPES FOR FREEZING SOIL 1

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Results are given for the computed artificial freezing of soil using low-temperature heat pipes on the basis of a very simple mathematical model.

Recent investigations [1, 2], both in the Soviet Union and abroad, have shown the promise of using heat pipes (HP) to cool soil – to lower the soil temperature at the expense of the outside air. By introducing heat-exchangers based on an evaporation–condensation cycle into the oil and gas industry, one would evidently resolve the fundamental problems of reliable operation of working bores in the permafrost region, the insulation of oil and gas lines in the same region, and could maintain the soil at constant temperature, avoiding thawing and structural breakdown.

We consider the action of a heat pipe located in the ground (e.g., by means of preliminary drilling of a narrow bore). Generally, ammonia is used as the working liquid. When there is a negative temperature drop between the above-ground and the below-ground parts of the heat pipe, i.e., in winter, when the air temperature is below the soil temperature, there is a flux of heat from the ground going to evaporation of the heat-transfer agent, followed by freezing of the soil. The vapor of the working liquid condenses in the upper (above-ground) part of the heat pipe, and then the condensate moves downwards along the heat pipe wick under the action of capillary forces and gravity. Heat transfer in a direction opposite to that of gravity allows one to use closed evaporative thermal siphons as evaporation–condensation devices. Thermal siphons have higher thermal resistance than heat pipes; their main advantage is in simplicity of manufacture. When there is a reverse temperature drop, in winter, the heat-transfer process in the heat pipe is stopped. Now heat transfer occurs in the reverse direction due to conduction along the heat pipe wall, but this is insignificant. Thus, when there are negative air temperatures around the part of the heat pipe located in the soil, there is a frozen zone, which does not fully thaw during the relatively short-duration season of positive temperatures (roughly 3 months).

The basic processes occurring with artificial freezing of soil by means of low-temperature heat pipes are: 1) freezing (or thawing) of the soil when it interacts with the atmosphere:2) heat and mass transfer in the heat pipe; 3) freezing of the soil by means of the heat pipe.

Each of these special problems is independently of interest and each is rather complex. To solve the complete problem we need to consider processes 1)-3) simultaneously, i.e., we must formulate the problem as a combined one. In addition, in solving problems 1)-3) we need to take into account the following situation. Because of the small heat-transfer coefficient between the heat pipe surface and the air, the area of the heat-pipe condenser is the element governing the efficiency of the heat pipe. Therefore, one must take cognizance of the fact that the surface of the above-ground part of the heat pipe can be increased, e.g., with the help of fins. Finning of the heat-pipe surface is also possible in the upper part of the soil, where the temperature during winter is considerably below that of the immersed end of the heat pipe.

We shall briefly describe the formulation of problems 1)-3). The theoretical investigation of the temperature field in the ground (during freezing or thawing) is done by solving a Stefan-type problem. The ground, which consists of frozen and unfrozen zones, is considered as a two-layer system, in which the heat transfer is described by the ordinary heat-conduction equations. At the boundary of the zones the Stefan condition is given. The remaining boundary conditions are usually taken to be type 1 or type 3 boundary conditions at the ground–air boundary, the temperature of the neutral ground layer, and an arbitrary temperature distribution at the initial time.

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The heat and mass transfer in the heat pipe is investigated to determine the temperature distribution at the inner wall of the evaporator part of the heat pipe, i.e., the part of the heat pipe located in the ground. The boundary condition for the condenser (above ground) part of the heat pipe is of type 1 or type 3. As a boundary condition in the evaporator part of the heat pipe we take a type 4 boundary condition.

Finally, the problem of freezing of the soil is again a Stefan-type problem. As the boundary conditions, in addition to the Stefan condition and the condition at the heat pipe–soil boundary, we take the solution of problem 1 and the temperature of the neutral ground layer.

As a first approximation we consider the following model:

1) the air temperature is assumed constant, less than zero (T_a = const < 0);

2) the soil temperature is taken to be the melting temperature (T_s = 0°C).

The heat-conduction equation for the frozen zone has the form

$$\frac{\partial T_f}{\partial t} = a_f\left(\frac{1}{r}\frac{\partial T_f}{\partial r} + \frac{\partial^2 T_f}{\partial r^2}\right), \quad r_0 \leqslant r \leqslant \zeta(t). \tag{1}$$

The boundary conditions are

$$\lambda_f \frac{\partial T_f}{\partial r}\bigg|_{r=r_0} = \alpha(T_f|_{r=r_0} - T_a), \quad T_f|_{r=\zeta(t)} = 0, \tag{2}$$

where $\alpha = [1/(\alpha_{conv}S_{co}) + 1/(\alpha_{cond}S_{ci}) + 1/(\alpha_{evap}S_{evi})]^{-1}S_{evo}^{-}$; α_{conv}, α_{cond}, and α_{evap} are determined from the known correlations; and we neglect the thermal resistances of the heat pipe body and the heat pipe–soil contact. The condition at the frozen boundary (the Stefan condition) is

$$\lambda_f \frac{\partial T_f}{\partial r}\bigg|_{r=\zeta} = lw\frac{d\zeta}{dt}, \quad \zeta|_{t=0} = r_0. \tag{3}$$

We now write the problem of Eqs. (1)-(3) in dimensionless form. To do this we introduce the following dimensionless variables:

$$\bar{T}_f = \frac{T_f}{T_a}, \quad \bar{r} = \frac{r}{r_0}, \quad \bar{\zeta} = \frac{\zeta}{r_0}, \quad \bar{t} = \frac{t}{t_0}, \quad t_0 = \frac{lwr_0^2}{\lambda_f T_a}.$$

Substituting these variables into Eq. (1) and the boundary conditions (2) and (3), we rewrite them in the form (omitting the bars)

$$\varepsilon\frac{\partial T_f}{\partial t} = \frac{1}{r}\frac{\partial T_f}{\partial r} + \frac{\partial^2 T_f}{\partial r^2}, \quad 1 \leqslant r \leqslant \zeta(t), \tag{4}$$

$$\frac{\partial T_f}{\partial r}\bigg|_{r=1} = \gamma(T_f|_{r=1} - 1), \quad T_f|_{r=\zeta(t)} = 0, \tag{5}$$

$$\frac{\partial T_f}{\partial r}\bigg|_{r=\zeta} = \frac{d\zeta}{dt}, \quad \zeta|_{t=0} = 1, \tag{6}$$

where $\varepsilon = \lambda_f T_a/a_f lw$; $\gamma = \alpha r_0/\lambda_f$.

Since the quantity ε is small ($|\varepsilon| < 0.5$) for a wide range of values of λ_f, T_a, a_f, and w, we seek a solution of the problem of Eqs. (4)-(6) in the form of the expansion

$$T_f = T_f^{(0)} + \varepsilon T_f^{(1)} + \varepsilon^2 T_f^{(2)} + \ldots \tag{7}$$

Substituting Eq. (7) into Eq. (4) and the boundary conditions (5), and equating powers of ε, we obtain the following problems for the zeroth-order and first-order approximations:

$$\frac{1}{r}\frac{\partial T_f^{(0)}}{\partial r} + \frac{\partial^2 T_f^{(0)}}{\partial r^2} = 0, \tag{8}$$

$$\left.\frac{\partial T_f^{(0)}}{\partial r}\right|_{r=1} = \gamma\left(T_f^{(0)}|_{r=1} - 1\right), \quad T_f^{(0)}|_{r=\zeta(t)} = 0,$$

$$\frac{\partial T_f^{(0)}}{\partial t} = \frac{1}{r}\frac{\partial T_f^{(1)}}{\partial r} + \frac{\partial^2 T_f^{(1)}}{\partial r^2}, \tag{9}$$

$$\left.\frac{\partial T_f^{(1)}}{\partial r}\right|_{r=1} = \gamma T_f^{(1)}|_{r=1}, \quad T_f^{(1)}|_{r=\zeta(t)} = 0.$$

The solution of boundary problem (8) has the form

$$T_f^{(0)} = -\frac{\gamma}{\gamma \ln \zeta + 1} \ln \frac{r}{\zeta}. \tag{10}$$

Integrating Eq. (9) and determining the constants of integration from the appropriate boundary conditions, we obtain

$$T_f^{(1)} = \frac{\gamma\dot{\zeta} r^2 (1 - \gamma + \gamma \ln r)}{4\zeta(\gamma \ln \zeta + 1)^2} + C_1 \ln r + C_2;$$

$$C_1 = -\frac{\gamma\dot{\zeta}}{4(1 + \gamma \ln \zeta)^3}\left[\frac{2 - 2\gamma + \gamma^2}{\zeta} + \zeta(1 - \gamma + \gamma \ln \zeta)\right]; \tag{11}$$

$$C_2 = -\frac{\gamma\dot{\zeta}\zeta(1 - \gamma + \gamma \ln \zeta)}{4(1 + \gamma \ln \zeta)^3} - C_1 \ln \zeta; \quad \dot{\zeta} \equiv \frac{d\zeta}{dt}.$$

Limiting ourselves to the first two terms of the expansion, we can write

$$T_f \simeq T_f^{(0)} + \varepsilon T_f^{(1)}. \tag{12}$$

Hence we find

$$\left.\frac{\partial T_f}{\partial r}\right|_{r=\zeta} = \frac{\varepsilon\dot{\zeta}\gamma}{4(\gamma \ln \zeta + 1)^3}\left[2(\gamma \ln \zeta + 1)^2 - 2\gamma(\gamma \ln \zeta + 1) + \gamma^2 - \frac{2 - 2\gamma + \gamma^2}{\zeta}\right] - \frac{\gamma}{\zeta(\gamma \ln \zeta + 1)}. \tag{13}$$

Therefore, the Stefan condition (6), taking account of Eq. (13), takes the form

$$\dot{\zeta}\left\{\frac{\varepsilon\gamma}{4(\gamma \ln \zeta + 1)^3}\left[2(\gamma \ln \zeta + 1)^2 - 2\gamma(\gamma \ln \zeta + 1) + \gamma^2 - (2 - 2\gamma + \gamma^2)/\zeta^2\right] - 1\right\} = \frac{\gamma}{\zeta(\gamma \ln \zeta + 1)}. \tag{14}$$

Its solution, satisfying the boundary condition $\zeta = 1$ at $t = 0$, has the form

$$t = \frac{\varepsilon\zeta^2}{4}\left(1 - \frac{\gamma}{\gamma \ln \zeta + 1}\right) + \frac{\varepsilon(2 - 2\gamma + \gamma^2)}{4\gamma(\gamma \ln \zeta + 1)} - \frac{\zeta^2\left(2\frac{\gamma \ln \zeta + 1}{\gamma} - 1\right)}{4} - \frac{\gamma - 2}{4\gamma}(1 - \varepsilon). \tag{15}$$

Thus, an approximate formula has been obtained to estimate the growth rate of the radius of freezing around the heat pipes. In a subsequent paper the restrictions 1 and 2 on the temperature of the air and ground are removed and the problem of artificial freezing is solved in a more general case.

NOTATION

T, temperature, °C; t, time, sec; λ, thermal conductivity, W/m·deg; r, radius, m; α, heat-transfer coefficient, W/m^2·deg; a, thermal diffusivity, m^2/sec; ζ, distance of the freezing front from the heat pipe axis, m; S, surface area, m^2; l, heat of crystallization, J/kg; w, humidity of the soil, kg/m^3. Subscripts: a, air; f, frozen; o, outer; i, inner; ev, evaporator; c, condenser; conv, convection; evap, evaporation; cond, condensation.

LITERATURE CITED

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